Scattering Matrix of Dielectric Resonator Coupled with Two Microstrip Lines

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Abstract **—The coupling between a dipolar $TE_{01\delta}$ cylindrical dielectric resonator and two microstrip lines terminated by different loads is modeled by a lumped equivalent circuit. A three-dimensional finite element method is used to determine the resonant frequency, quality factor, and coupling coefficients as a function of the electromagnetic parameters of the structure.**

I. Introduction

Dielectric resonators (DR's) are now commonly used in many microwave devices, among them filters and oscillators. In some of these applications, cylindrical DR's in their dipolar $TE_{01\delta}$ mode are coupled to two microstrip lines (Fig. 1) and shielded in a metallic box. A number of methods have been proposed to characterize such structures. But the analyses published [1]–[6] on this subject assume that

- the cylindrical DR is enclosed in a cylindrical box so that the structure is axisymmetrical;
- the presence of the lines near the DR is not considered;
- the lines are shorted, opened, or matched.

In this paper, we propose a new approach, using a three-dimensional finite element method (3-D FEM) to establish a lumped equivalent circuit of the DR device and to evaluate its scattering matrix parameters. The DR and the lines terminated by any load are housed in a parallelepiped metallic enclosure. Experimental results are also given to verify theoretical ones.

To analyze this microwave DR device, the following assumptions are made:

- the unloaded quality factor of the DR is high;
- only the dominant mode $TE_{01\delta}$ is present in the structure;
- the parameter values of the equivalent circuit are constant around the resonant frequency of the shielded $TE_{01\delta}$ DR mode.

II. Circuit Analysis

The DR acting on its $TE_{01\delta}$ mode is placed on a microstrip lines substrate so that its magnetic field lines link those of the lines (Fig. 2). The DR is represented by a simple parallel resonant (L_r, C_r, R_r) circuit. We assume that coupling with the two lines occurs only through two mutual inductances, M_1 and M_2. The original electric parameters of

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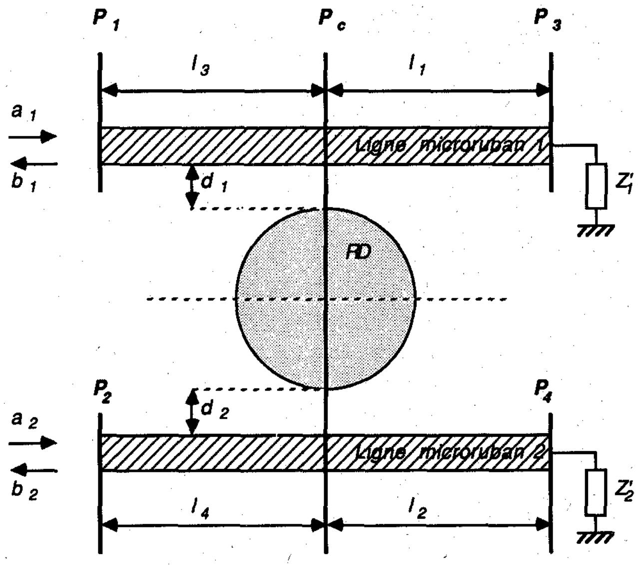

Fig. 1. Coupling between a DR and two loaded microstrip lines.

the lines and the DR are modified by each other and by the presence of the metallic enclosure. In the next section, we will see that these effects are taken into account by using a 3-D analysis of the structure.

To establish a lumped equivalent circuit of the structure given in Fig. 1, we define the coupling plane, P_c, and the reference planes P_1 and P_2. The latter are located in the planes where the test setup is connected. The scattering matrix coefficients will be expressed and measured in these planes. Also defined are reference planes P_3 and P_4, where the output impedances Z_1 and Z_2 are connected. In P_c, these impedances are respectively noted Z_1' and Z_2', with

$$Z_i' = Z_0 \frac{Z_i + jZ_0 \tan \beta l_i}{Z_0 + jZ_i \tan \beta l_i}, \qquad i = 1,2 \tag{1}$$

where Z_0 is the characteristic impedance of the microstrip lines and β is their phase constant.

We note, as shown in Fig. 2, a phase inversion of the field on the two lines [2]. An ideal transformer introduced in the equivalent circuit will simulate this effect without affecting the circuit symmetry. With these considerations, the lumped element circuit configuration of the system presented in Fig. 1 is given in Fig. 3.

To calculate the S_{ij} parameters, we name ω_0 the resonant frequency of the structure, Q_0 is its unloaded quality factor, and α_1 and α_2 are the coupling coefficients between the DR

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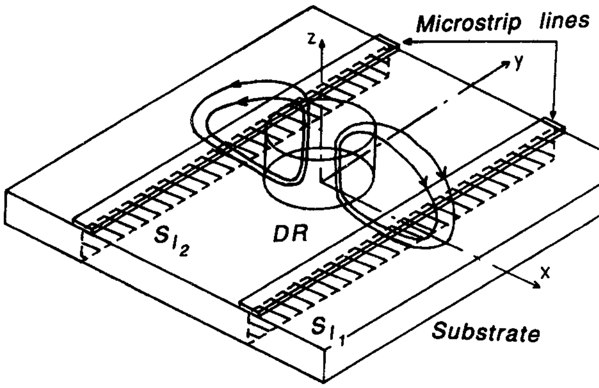

Fig. 2. Magnetic field lines of the DR around the microstrip lines.

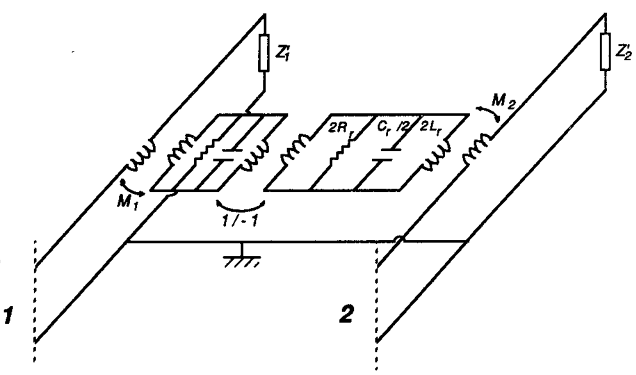

Fig. 3. Lumped equivalent circuit of the coupling between the DR and two microstrip lines.

and the two lines. These parameters are defined as follows:

$$\alpha_i = \frac{Q_0 \omega_0}{2Z_0} \frac{M_i^2}{L_r}, \qquad i = 1,2 \tag{2}$$

where M_i^2/L_r is related to distance d_i.

A simple analysis of the electrical model yields S_{ij} parameters in the reference planes P_1 and P_2 as a function of ω, ω_0, Q_0, Z_0, α_1, α_2, Z_1', Z_2', l_3, and l_4. Expressions are given in the Appendix.

III. Field Analysis

In the previous section, it was shown that the characterization of coupling between a DR in the $TE_{01\delta}$ mode and two microstrip lines requires the following parameters: the resonant frequency, ω_0; the unloaded quality factor, Q_0, of the DR shielded in its structure; and the coupling coefficients α_1 and α_2. These parameters are determined by the FEM applied to solve the vectorial wave equation (3) [7]–[10]:

$$\nabla \times \left(\frac{1}{\epsilon_i} \nabla \times \vec{H} \right) - k_0^2 \cdot \vec{H} = -j\omega\epsilon_0 \sum_{k=1}^{4} \overrightarrow{J_{mp_k}} \tag{3}$$

where $\vec{H}$ is the magnetic field distribution, ϵ_i is the relative permittivity of homogeneous, linear, lossless isotropic medium i, $k_0 = \omega\sqrt{\epsilon_0 \mu_0}$, and $\overrightarrow{J_{mp_k}}$ is the magnetic surface distribution current in reference plane P_k, with $k = 1,4$. $\overrightarrow{J_{mp_k}}$ is related to the amplitude coefficients of the incident wave a_k and the reflected wave b_k (Fig. 4).

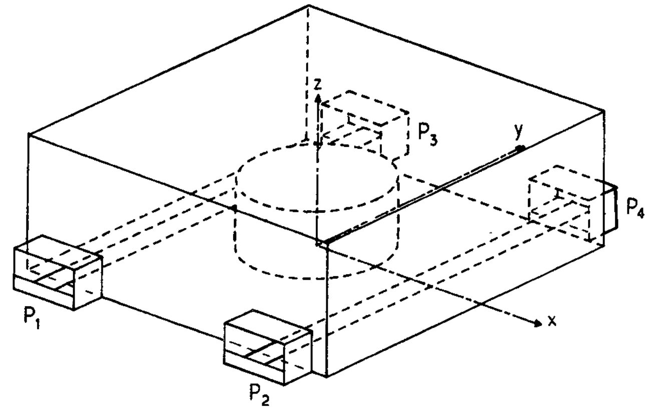

Fig. 4. The three-dimensional structure. For the DR, radius R_r = 8 mm, height H_r = 6.1 mm, permittivity ϵ_r = 37.1, and $F_0(\tan\delta)^{-1}$ = 40000. For the microstrip lines, substrate thickness h_s = 0.83 mm, metallic armature width ω_s = 2.39 mm, and permittivity ϵ_r = 2.53. For the enclosure, basis dimension, $X_c \times Y_c = 40 \times 40$ mm^2, height H_c = 10 mm, and conductivity $\sigma = 1.57 \times 10^7$ mho/m.

Taking into account boundary conditions, we obtain, using the FEM, the following global matrix equation:

$$\left[[A] - k_0^2[B]\right]\{H\} = \sum_{k=1}^{4} (a_k + b_k)\{J_{s_k}\} \tag{4}$$

where $[A]$ and $[B]$ are matrices related to first and second terms of (3), and $\{J_{s_k}\}$ is a vector related to the excitation of the structure in reference plane P_k.

Analysis of scattering matrix parameters' response as a function of frequency yields the parameters involved in the lumped equivalent expressions (A1) and (A2). The three-dimensional forced oscillation problem thus formed, unfortunately, requires large computation time (for example, one hour of CPU time (on HP 9000/835) for the computation of one frequency on a 1200 nodes mesh). The parameters ω_0, Q_0, and α_i, however, can be determined by considering the free oscillation problem. The second member of (4) then cancels, as $\overrightarrow{J_{mp_k}} = 0$. The expression $k^2 = k_0^2 = \omega_0^2\sqrt{\epsilon_0\mu_0}$ becomes the eigenvalue function of that equation. Solving (4), we obtain the resonant frequency, ω_0, and the magnetic field repartition of each resonance mode of the structure. These parameters are used to compute the total energy stored in the structure, $\overline{W}$, the metallic and dielectric losses, $\overline{P}$, and then the unloaded quality factor, Q_0, which can be written

$$Q_0 = \omega_0 \frac{\overline{W}}{\overline{P}}. \tag{5}$$

We must now determine the coupling coefficients α_1 and α_2 between the DR and the lines. These parameters satisfy [4], [5]

$$\alpha_i = \frac{Q_0\omega_0}{2Z_0}\mu_0^2 \frac{\left[\iint_{S_{li}} \vec{H} \cdot \overrightarrow{dS_{li}}\right]^2}{2\overline{W}}. \tag{6}$$

The quantity

$$\phi = \iint_{S_{li}} \vec{H} \cdot \overrightarrow{dS_{li}}$$

represents the magnetic flux through the section S_{li} under

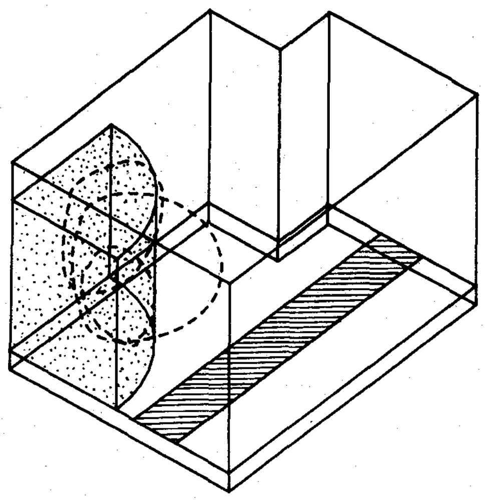

Fig. 5. Magnetic field lines of a shielded DR including substrate and upper armature of two microstrip lines.

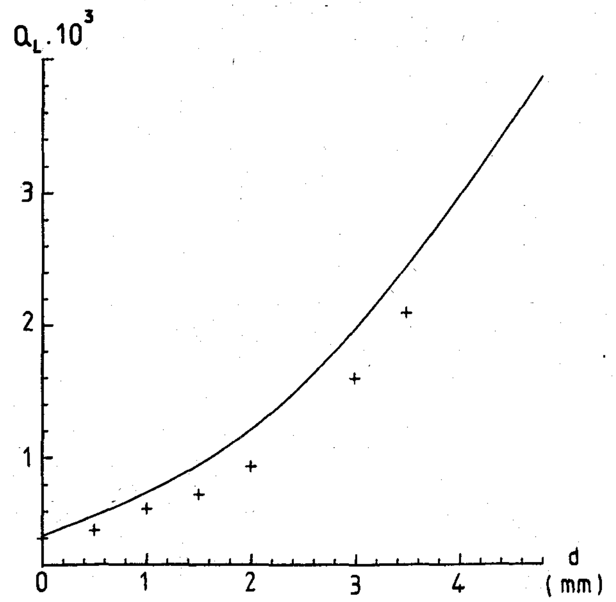

Fig. 6. External quality factor variations for a DR coupled with a straight microstrip line: —— theoretical results; + experimental results.

the microstrip lines (Fig. 2). As the magnetic field is known at any point of the composite dielectric structures, the FEM analysis makes it possible to compute α_1 and α_2 as a function of the distances d_1 and d_2 between the DR and the microstrip lines. This 3-D FEM analysis permits taking into account the effect of the presence of the microstrip lines near the DR on the resonance frequency and the field repartition [10]. For example, we have studied the structure shown in Fig. 5. Symmetries of the $TE_{01\delta}$ mode are exploited to reduce the amount of computation time required; only a quarter of the general structure is considered. Enclosure planes are assumed to be electric walls. Magnetic field lines have been drawn to identify the dipolar mode. For the dimensions given in Fig. 4, we obtain $f_0 = 5.324$ GHz and $Q_0 = 5800$.

Fig. 6 shows variations in both experimental and theoretical loaded quality factors, $Q_L = Q_0/(1+\alpha_1)$ [1], when the distance between a straight line and the DR changes.

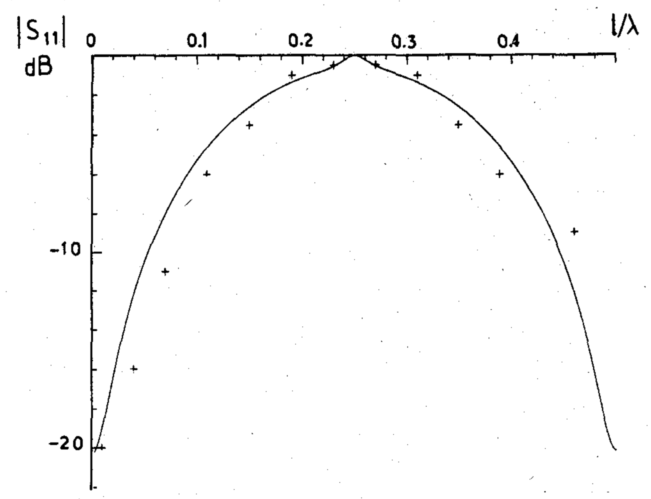

Fig. 7. Scattering parameter modulus $|S_{11}|$ as a function of the distance between the coupling plane P_c and reference planes P_3 and P_4: $Z_1 = Z_2 = 0$; $d_1 = d_2 = 0$; $\alpha_1 = \alpha_2 = 22$; —— theoretical results; + experimental results.

Fig. 7 compares experimental and theoretical variations of $|S_{11}|$ as a function of the distance between the coupling plane, P_c, and the reference planes P_3 and P_4 when $d_1 = d_2 = 0$, $l_1 = l_2 = l$, and $Z_1 = Z_2 = 0$, so that $Z_1' = Z_2' = jZ_0 \tan \beta l$. We can verify that the incident power transmitted to the output line will be a maximum when $l = k\lambda/2$; this power cancels for $l = (2k+1)\lambda/4$, $k = 0, 1, 2, \cdots$.

IV. Conclusion

A lumped equivalent circuit has been proposed to model the coupling of a DR with two microstrip lines terminated by any impedance. Scattering parameters have been computed. Methods have been presented to evaluate these coefficients from a field analysis. They are shown to give good agreement between computed and experimental results.

Appendix

The scattering matrix parameters are

$$S_{11} = \left\{ \frac{1}{D} \left[\left(\sqrt{\frac{\alpha_2}{\alpha_1}} - \sqrt{\frac{\alpha_1}{\alpha_2}} \right) \left(1 - \frac{2\omega^2}{\omega_0^2} \right) + \frac{2}{\sqrt{\alpha_1 \alpha_2}} \left(-1 + \frac{(Z_1' - Z_2')}{Z_0} + \frac{Z_1' Z_2'}{Z_0^2} \right) - \sqrt{\alpha_1 \alpha_2} \frac{2\omega^2}{Q_0^2 \omega_0^2} - \left(\frac{Z_1'}{Z_0} \sqrt{\frac{\alpha_2}{\alpha_1}} + \frac{Z_2'}{Z_0} \sqrt{\frac{\alpha_1}{\alpha_2}} \right) \left(1 + \frac{2\omega^2}{\omega_0^2} \right) \right] + \frac{j}{D} \left[\frac{2\omega}{Q_0 \omega_0} \left(\sqrt{\frac{\alpha_2}{\alpha_1}} \left(1 - \frac{Z_1'}{Z_0} \right) - \sqrt{\frac{\alpha_1}{\alpha_2}} \left(1 + \frac{Z_2'}{Z_0} \right) \right) - \frac{2\omega^3}{Q_0 \omega_0^3} \sqrt{\alpha_1 \alpha_2} + \frac{2Q_0}{\sqrt{\alpha_1 \alpha_2}} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \cdot \left(1 + \frac{Z_1' - Z_2'}{Z_0} + \frac{Z_1' Z_2'}{Z_0^2} \right) \right] \right\} e^{-2j\beta l_3} \qquad \text{(A1)}$$

$$S_{21} = -\frac{2}{D} e^{-j\beta(l_4 - l_3)} \qquad \text{(A2)}$$

where

$$D=\Bigg[-\left(\sqrt{\frac{\alpha_2}{\alpha_1}}+\sqrt{\frac{\alpha_1}{\alpha_2}}\right)\left(1+\frac{2\omega^2}{\omega_0^2}\right)+\left(\frac{Z_2'}{Z_0}\sqrt{\frac{\alpha_1}{\alpha_2}}+\frac{Z_1'}{Z_0}\sqrt{\frac{\alpha_2}{\alpha_1}}\right)\left(\frac{2\omega^2}{\omega_0^2}-1\right)+\frac{2\omega^2}{Q_0^2\omega_0^2}\sqrt{\alpha_1\alpha_2}+\frac{2}{\sqrt{\alpha_1\alpha_2}}\left(1+\frac{(Z_1'+Z_2')}{Z_0}+\frac{Z_1'Z_2'}{Z_0^2}\right)+j\Bigg[-\frac{2\omega}{Q_0\omega_0}\left(\sqrt{\frac{\alpha_2}{\alpha_1}}\left(1+\frac{Z_1'}{Z_0}\right)+\sqrt{\frac{\alpha_1}{\alpha_2}}\left(1+\frac{Z_2'}{Z_0}\right)\right)-\frac{2\omega^3}{Q_0\omega_0^3}\sqrt{\alpha_1\alpha_2}+\frac{2Q_0}{\sqrt{\alpha_1\alpha_2}}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)\cdot\left(1+\frac{Z_1'+Z_2'}{Z_0}+\frac{Z_1'Z_2'}{Z_0^2}\right)\Bigg]\Bigg]. \tag{A3}$$

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